Graduate Public Finance: Efficiency of Taxation

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1. Marshallian surplus

2. Path dependence problem and income effects

3. Definitions of EV, CV, and excess burden with income effects

4. Harberger Approximation

5. Exact Consumer Surplus (Hausman 1981)

6. Empirical Applications

7. Welfare Analysis in Behavioral Models
Incidence: effect of policies on distribution of economic pie

Efficiency or deadweight cost: effect of policies on size of the pie

Focus in efficiency analysis is on quantities, not prices
Government raises taxes for one of two reasons:

1. To raise revenue to finance public goods
2. To redistribute income

But to generate $1 of revenue, welfare of those taxed falls by more than $1 because the tax distorts behavior.

How to implement policies that minimize these efficiency costs?

Start with positive analysis of how to measure efficiency cost of a given tax system.
Simplest analysis of efficiency costs: Marshallian surplus

Two assumptions:

1. Quasilinear utility: no income effects, money metric
2. Competitive production
Partial Equilibrium Model: Setup

- Two goods: $x$ and $y$

- Consumer has wealth $Z$, utility $u(x) + y$, and solves

$$\max_{x,y} u(x) + y \text{ s.t. } (p + \tau)x(p + \tau, Z) + y(p + \tau, Z) = Z$$

- Firms use $c(S)$ units of the numeraire $y$ to produce $S$ units of $x$

- Marginal cost of production is increasing and convex:

$$c'(S) > 0 \text{ and } c''(S) \geq 0$$

- Firm’s profit at pretax price $p$ and level of supply $S$ is

$$pS - c(S)$$
With perfect optimization, supply fn for $x$ is implicitly defined by the marginal condition:

$$p = c'(S(p))$$

Let $\eta_S = p \frac{S'}{S}$ denote the price elasticity of supply.

Let $Q$ denote equilibrium quantity sold of good $x$.

$Q$ satisfies:

$$Q(\tau) = D(p + \tau) = S(p)$$

Consider effect of introducing a small tax $d\tau > 0$ on $Q$ and surplus.
Excess Burden of Taxation

Price

$30.0

Quantity

1500

D

S

A

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Excess Burden of Taxation

Price

$36.0

$30.0

Quantity

1350

1500

S+t

S

D

Excess Burden

$\$t$

$\$t$

$\$t$

Price

Quantity

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Efficiency Cost: Qualitative Properties

1. Excess burden increases with square of tax rate
2. Excess burden increases with elasticities
EB Increases with Square of Tax Rate
EB Increases with Square of Tax Rate

\[P\]

\[Q\]

\[D\]

\[S\]

\[S+t_1\]

\[P_1\]

\[P_2\]

\[$t_1\]

\[Q_1\]

\[Q_2\]

\[C\]

\[A\]

\[B\]
EB Increases with Square of Tax Rate

Change in EB

$\Delta t_2$ $\Delta P$

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Comparative Statics

(a) Inelastic Demand

(b) Elastic Demand

\[ P_1 \leq P_2 \]

\[ Q_1 \leq Q_2 \]

\[ S^t \]

\[ D \]

\[ S \]

\[ B \]

\[ C \]

\[ A \]

\[ P \]

\[ Q \]

Comparative Statics

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Part 3: Efficiency
Tax Policy Implications

With many goods, the most efficient way to raise tax revenue is:

1. Tax inelastic goods more (e.g. medical drugs, food)
2. Spread taxes across all goods to keep tax rates relatively low on all goods (broad tax base)

These are two countervailing forces; balancing them requires quantitative measurement of excess burden
Measuring Excess Burden: Marshallian Surplus

How to measure excess burden? Three empirically implementable methods:

1. In terms of supply and demand elasticities

2. In terms of total change in equilibrium quantity caused by tax

3. In terms of change in government revenue
Method 1: Supply and Demand Elasticities

\[ EB = -\frac{1}{2}dQd\tau \]

\[ EB = -\frac{1}{2}S'(p)dpd\tau = (1/2)(pS'/S)(S/p)\frac{\eta_D}{\eta_S - \eta_D}d\tau^2 \]

\[ EB = -\frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D}pQ\left(\frac{d\tau}{p}\right)^2 \]

- Note: second line uses incidence formula \( dp = \left(\frac{\eta_D}{\eta_S - \eta_D}\right)d\tau \)
- Tax revenue \( R = Qd\tau \)
- Useful expression is deadweight burden per dollar of tax revenue:

\[ \frac{EB}{R} = -\frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} \frac{d\tau}{p} \]
Method 2: Distortions in Equilibrium Quantity

- Define $\eta_Q = -\frac{dQ}{d\tau} \frac{p_0}{Q}$

- $\eta_Q$: effect of a 1% increase in price via a tax change on equilibrium quantity, taking into account the endogenous price change

- This is the coefficient $\beta$ in a reduced-form regression:

  \[ \log Q = \alpha + \beta \frac{\tau}{p_0} + \epsilon \]

- Identify $\beta$ using exogenous variation in $\tau$. Then:

  \[ EB = -(1/2) \frac{dQ}{d\tau} d\tau d\tau \]
  \[ = -(1/2) \frac{dQ}{d\tau} \left( \frac{p}{Q} \right) \left( \frac{Q}{p} \right) d\tau d\tau \]
  \[ = (1/2) \eta_Q pQ \left( \frac{d\tau}{p} \right)^2 \]
Marginal Excess Burden of Tax Increase

- Excess burden of a tax \( \tau \) is
  \[
  EB(\tau) = -(1/2) \frac{dQ}{d\tau} \tau^2
  \]

- Consider EB from raising tax by \( \Delta \tau \) given pre-existing tax \( \tau \):
  \[
  EB(\Delta \tau) = -(1/2) \frac{dQ}{d\tau} \left[ (\tau + \Delta \tau)^2 - \tau^2 \right]
  \]
  \[
  = -(1/2) \frac{dQ}{d\tau} \cdot [2\tau \cdot \Delta \tau + (\Delta \tau)^2]
  \]
  \[
  = -\tau \frac{dQ}{d\tau} \Delta \tau - (1/2) \frac{dQ}{d\tau} (\Delta \tau)^2
  \]

- First term is first-order in \( \Delta \tau \); second term is second-order \( ((\Delta \tau)^2) \)

- This is why taxing markets with pre-existing taxes generates larger marginal EB

- EB of \( \Delta \tau = 1\% \) is 10 times larger if \( \tau = 10\% \) than if \( \tau = 0 \).
First vs. Second-Order Approximations

- Computing marginal excess burden by differentiating formula for excess burden gives:

\[
\frac{dEB}{d\tau} \cdot \Delta\tau = -\tau \frac{dQ}{d\tau} \cdot \Delta\tau
\]

- First derivative of \( EB(\tau) \) only includes first-order term in Taylor expansion:

\[
EB(\tau + \Delta\tau) = EB(\tau) + \frac{dEB}{d\tau} \Delta\tau + \frac{1}{2} \frac{d^2EB}{d\tau^2} (\Delta\tau)^2
\]

- First-order approximation is accurate when \( \tau \) large relative to \( \Delta\tau \)

  - Ex: \( \tau = 20\% \), \( \Delta\tau = 5\% \) implies first term accounts for 90% of EB

  - But introduction of new tax \((\tau = 0)\) generates EB only through second-order term
Method 3: Leakage in government revenue

- To first order, marginal excess burden of raising \( \tau \) is:
  \[
  \frac{\partial EB}{\partial \tau} = -\tau \frac{dQ}{d\tau}
  \]

- Observe that tax revenue \( R(\tau) = Q\tau \)
  - Mechanical revenue gain: \( \frac{\partial R}{\partial \tau} \bigg|_{Q} = Q \)
  - Actual revenue gain: \( \frac{\partial R}{\partial \tau} = Q + \tau \frac{dQ}{d\tau} \)

- MEB is the difference between mechanical and actual revenue gain:
  \[
  \frac{\partial R}{\partial \tau} \bigg|_{Q} - \frac{dR}{d\tau} = Q - \left[ Q + \tau \frac{dQ}{d\tau} \right] = -\tau \frac{dQ}{d\tau} = \frac{\partial EB}{\partial \tau}
  \]
First vs. Second-Order Approximations

- Why does leakage in govt. revenue only capture first-order term?
  - Govt revenue loss: rectangle in Harberger trapezoid, proportional to $\Delta \tau$
  - Consumer and producer surplus loss: triangles in trapezoid (proportional to $\Delta \tau^2$)

- Method 3 is accurate for measuring marginal excess burden given pre-existing taxes but not introduction of new taxes
Excess Burden of a Tax Increase: Harberger Trapezoid

- Lost govt. revenue (1st order)
- Excess Burden of a Tax Increase: Harberger Trapezoid
- Lost cons. surplus (2nd order)
- Lost producer surplus (2nd order)
General Model with Income Effects

- Drop quasilinearity assumption and consider an individual with utility
  \[ u(c_1, \ldots, c_N) = u(c) \]

- Individual’s problem:
  \[
  \max_c u(c) \quad \text{s.t.} \quad q \cdot c \leq Z
  \]
  where \( q = p + \tau \) denotes vector of tax-inclusive prices and \( Z \) is wealth

- Labor can be viewed as commodity with price \( w \) and consumed in negative quantity
Demand Functions and Indirect Utility

- Let \( \lambda \) denote multiplier on budget constraint

- First order condition in \( c_i \):

  \[ u_{c_i} = \lambda q_i \]

- These conditions implicitly define:
  
  - \( c_i(q, Z) \): the Marshallian ("uncompensated") demand function
  
  - \( v(q, Z) \): the indirect utility function
Measuring Deadweight Loss with Income Effects

- Question: how much utility is lost because of tax beyond revenue transferred to government?

- Marshallian surplus does not answer this question with income effects
  - Problem: not derived from utility function or a welfare measure
  - Creates various problems such as “path dependence” with taxes on multiple goods
    \[
    \Delta CS(\tau^0 \to \tilde{\tau}) + \Delta CS(\tilde{\tau} \to \tau^1) \neq \Delta CS(\tau^0 \to \tau^1)
    \]
- Need units to measure “utility loss”
  - Introduce expenditure function to translate the utility loss into dollars (money metric)
Expenditure Function

- Fix utility at $U$ and prices at $q$
- Find bundle that minimizes cost to reach $U$ for $q$:
  \[ e(q, U) = \min_c q \cdot c \text{ s.t. } u(c) \geq U \]
- Let $\mu$ denote multiplier on utility constraint
- First order conditions given by:
  \[ q_i = \mu u_{c_i} \]
- These generate Hicksian (or compensated) demand fns:
  \[ c_i = h_i(q, u) \]
- Define individual’s loss from tax increase as
  \[ e(q^1, u) - e(q^0, u) \]
- Single-valued function $\rightarrow$ coherent measure of welfare cost, no path dependence
But where should $u$ be measured?

Consider a price change from $q^0$ to $q^1$

Utility at initial price $q^0$:

$$u^0 = v(q^0, Z)$$

Utility at new price $q^1$:

$$u^1 = v(q^1, Z)$$

Two concepts: compensating ($CV$) and equivalent variation ($EV$) use $u^0$ and $u^1$ as reference utility levels
Compensating Variation

- Measures utility at initial price level \( (u^0) \)

- Amount agent must be compensated in order to be indifferent about tax increase

\[
CV = e(q^1, u^0) - e(q^0, u^0) = e(q^1, u^0) - Z
\]

- How much compensation is needed to reach original utility level at new prices?

- \( CV \) is amount of ex-post cost that must be covered by government to yield same ex-ante utility:

\[
e(q^0, u^0) = e(q^1, u^0) - CV
\]
Equivalent Variation

- Measures utility at new price level

- Lump sum amount agent willing to pay to avoid tax (at pre-tax prices)

\[ EV = e(q^1, u^1) - e(q^0, u^1) = Z - e(q^0, u^1) \]

- \( EV \) is amount extra that can be taken from agent to leave him with same ex-post utility:

\[ e(q^0, u^1) + EV = e(q^1, u^1) \]
Efficiency Cost with Income Effects

- Goal: derive empirically implementable formula analogous to Marshallian EB formula in general model with income effects

- Literature typically assumes either
  1. Fixed producer prices and income effects
  2. Endogenous producer prices and quasilinear utility

- With both endogenous prices and income effects, efficiency cost depends on how profits are returned to consumers

- Formulas are very messy and fragile (Auerbach 1985, Section 3.2)
Efficiency Cost Formulas with Income Effects

- Derive empirically implementable formulas using Hicksian demand ($EV$ and $CV$)

- Assume $p$ is fixed → flat supply, constant returns to scale

- The envelope thm implies that $e_{q_i}(q, u) = h_i$, and so:

$$e(q^1, u) - e(q^0, u) = \int_{q^0}^{q^1} h(q, u) \, dq$$

- If only one price is changing, this is the area under the Hicksian demand curve for that good

- Note that optimization implies that

$$h(q, \nu(q, Z)) = c(q, Z)$$
Compensating vs. Equivalent Variation

\[ h(V(p_1, Z)) \]

\[ h(V(p_0, Z)) \]

\[ D \]

\[ x(p_1, Z) \]

\[ x(p_0, Z) \]
Compensating vs. Equivalent Variation

\[ h(V(p_1, Z)) \]
\[ h(V(p_0, Z)) \]

\[ EV \]

\[ x(p_1, Z) \]
\[ x(p_0, Z) \]

\[ D \]
Compensating vs. Equivalent Variation

\[ \text{Compensating Variation (CV)} = h(V(p_1, Z)) - h(V(p_0, Z)) \]

Graph:
- Price axis labeled \( p \)
- Quantity axis labeled \( x \)
- Points \( p_0 \) and \( p_1 \)
- Demand curve labeled \( D \)
- Areas shaded to represent CV
Marshallian Surplus

\[ h(V(p_1, Z)) \]

\[ h(V(p_0, Z)) \]

\[ x(p_1, Z) \]

\[ x(p_0, Z) \]
With one price change:

\[ EV < \text{Marshallian Surplus} < CV \]

But this is not true in general with multiple price changes because Marshallian Surplus is ill-defined.
Excess Burden

• Deadweight burden: change in consumer surplus less tax paid

• What is lost in excess of taxes paid?

• Two measures, corresponding to $EV$ and $CV$:

$$EB(u^1) = EV - (q^1 - q^0)h(q^1, u^1) \quad [Mohring \ 1971]$$

$$EB(u^0) = CV - (q^1 - q^0)h(q^1, u^0) \quad [Diamond \ and \ McFadden \ 1974]$$
\( h(V(p_0, Z)) \)

\( h(V(p_1, Z)) \)

\( x(p_1, Z) \)

\( x_C(p_1, V(p_0, Z)) \)

\( p \)

\( p_0 \)

\( p_1 \)

\( D \)

\( EB_{EV} \)

\( EB_{CV} \)
\[
D \times (h(V(p_0, Z)), h(V(p_1, Z))) \\
\sim \sim \sim \\
\text{Marshallian}
\]
In general, CV and EV measures of EB will differ

Marshallian measure overstates excess burden because it includes income effects

- Income effects are not a distortion in transactions
- Buying less of a good due to having less income is not an efficiency loss; no surplus foregone b/c of transactions that do not occur

CV = EV = Marshallian DWL only with quasilinear utility (Chipman and Moore 1980)
Implementable Excess Burden Formula

- Consider increase in tax $\tau$ on good 1 to $\tau + \Delta \tau$
- No other taxes in the system
- Recall the expression for $EB$:
  \[
  EB(\tau) = [e(p + \tau, U) - e(p, U)] - \tau h_1(p + \tau, U)
  \]
- Second-order Taylor expansion:
  \[
  MEB = EB(\tau + \Delta \tau) - EB(\tau) \\
  \approx \frac{dEB}{d\tau} \Delta \tau + \frac{1}{2} (\Delta \tau)^2 \frac{d^2EB}{d\tau^2}
  \]
Harberger Trapezoid Formula

\[
\frac{dEB}{d\tau} = h_1(p + \tau, U) - \tau \frac{dh_1}{d\tau} - h_1(p + \tau, U)
\]

\[
= -\tau \frac{dh_1}{d\tau}
\]

\[
\frac{d^2 EB}{d\tau^2} = -\frac{dh_1}{d\tau} - \tau \frac{d^2 h_1}{d\tau^2}
\]

- Standard practice in literature: assume \( \frac{d^2 h_1}{d\tau^2} = 0 \) (linear Hicksian); not necessarily well justified b/c it does not vanish as \( \Delta\tau \to 0 \)

\[
\Rightarrow MEB = -\tau \Delta\tau \frac{dh_1}{d\tau} - \frac{1}{2} \frac{dh_1}{d\tau} (\Delta\tau)^2
\]

- Formula equals area of “Harberger trapezoid” using Hicksian demands
Harberger Formula

- Without pre-existing tax, obtain “standard” Harberger formula:

\[ EB = -\frac{1}{2} \frac{dh_1}{d\tau} (\Delta \tau)^2 \]

- General lesson: use compensated (substitution) elasticities to compute \( EB \), not uncompensated elasticities

- To implement empirically, estimate Marshallian price elasticity and income elasticity. Then apply Slutsky eqn:

\[
\frac{\partial h_i}{\partial q_j} = \frac{\partial c_i}{\partial q_j} + c_j \frac{\partial c_i}{\partial Z} 
\]

\text{Hicksian Slope} \quad \text{Marshallian Slope} \quad \text{Income Effect}
Excess Burden with Taxes on Multiple Goods

- Previous formulas apply to case with tax on one good

- With multiple goods and fixed prices, excess burden of introducing a tax $\tau_k$

$$EB = -\frac{1}{2} \tau_k^2 \frac{dh_k}{d\tau_k} - \sum_{i \neq k} \tau_i \tau_k \frac{dh_i}{d\tau_k}$$

- Second-order effect in own market, first-order effect from other markets with pre-existing taxes

- Complementarity between goods important for excess burden calculations

- Ex: with an income tax, minimize total DWL tax by taxing goods complementary to leisure (Corlett and Hague 1953)
Show that ignoring cross effects by using one-good formula can be very misleading

Differentiate multiple-good Harberger formula w.r.t. $\tau_k$:

$$\frac{dEB}{d\tau_k} = -\tau_k \frac{dh_k}{d\tau_k} - \sum_{i \neq k} \tau_i \frac{dh_i}{d\tau_k}$$

If $\tau_k$ is small (e.g. gas tax), what matters is purely distortion in other markets, e.g. labor supply

As $\tau_k \rightarrow 0$, error in single-market formula approaches $\infty$
Hausman 1981: Exact Consumer Surplus

- Harberger formulas: empirically implementable but approximate

- Alternative approach: structural estimation of demand model

- Start by estimating Marshallian demand functions:
  \[ c(q, Z) = \gamma + \alpha q + \delta Z \]

- Then integrate to recover underlying indirect utility function \( v(q, Z) \)

- Invert to obtain expenditure function \( e(q, u) \) and compute “exact” EB

- Parametric approach: Hausman (AER 1981); non-parametric approach: Hausman and Newey (ECMA 1995)
Harberger vs. Hausman Approach

- Underscores broader difference between structural and quasi-experimental methodologies

- Modern literature focuses on deriving “sufficient statistic” formulas that can be implemented using quasi-experimental techniques

- Now develop general distinction between structural and sufficient statistic approaches to welfare analysis in a simple model of taxation
  - No income effects (quasilinear utility)
  - Constant returns to production (fixed producer prices)
  - But permit multiple goods (GE)
N goods: \( x = (x_1, \ldots, x_N) \); prices \((p_1, \ldots, p_N)\); wealth \( Z \)

- Normalize \( p_N = 1 \) \((x_N\) is numeraire\)
- Government levies a tax \( t \) on good 1
- Individual takes \( t \) as given and solves

\[
\max u(x_1, \ldots, x_{N-1}) + x_N \text{ s.t. } (p_1 + t)x_1 + \sum_{i=2}^{N} p_i x_i = Z
\]

To measure EB of tax, define social welfare as sum of individual’s utility and tax revenue:

\[
W(t) = \{ \max_x u(x_1, \ldots, x_{N-1}) + Z - (p_1 + t)x_1 - \sum_{i=2}^{N-1} p_i x_i \} + tx_1
\]

- Goal: measure \( \frac{dW}{dt} \) = loss in social surplus caused by tax change
\[ \begin{align*}
\omega_1 & \rightarrow \beta_1(t) \\
\omega_2 & \rightarrow \beta_2(t) \\
\vdots & \\
\omega_N & \rightarrow \frac{dW}{dt}(t)
\end{align*} \]

\( \omega \) = preferences, constraints

\( \beta = f(\omega, t) \)

\( y = \beta_1 X_1 + \beta_2 X_2 + \epsilon \)

dW/dt used for policy analysis

\( \omega \) not uniquely identified

\( \beta \) identified using program evaluation

Source: Chetty (2009)
Sufficient Statistics vs Structural Methods

- Structural method: estimate $N$ good demand system, recover $u$
  - Ex: use Stone-Geary or AIDS to recover preference parameters; then calculate “exact consumer surplus” as in Hausman (1981)

- Alternative: Harberger’s deadweight loss triangle formula

  - Private sector choices made to maximize term in red (private surplus)

    $$W(t) = \{ \max_{x} u(x_1, ..., x_{N-1}) + Z - (p_1 + t)x_1 - \sum_{i=2}^{N-1} p_i x_i \} + tx_1$$

  - Envelope conditions for $(x_1, ..., x_N)$ allow us to ignore behavioral responses $(\frac{dx_i}{dt})$ in term in red, yielding

    $$\frac{dW}{dt} = -x_1 + x_1 + t \frac{dx_1}{dt} = t \frac{dx_1}{dt}$$

    $$\rightarrow \frac{dx_1}{dt}$$ is a “sufficient statistic” for calculating $\frac{dW}{dt}$
Heterogeneity

- Benefit of suff stat approach particularly evident with heterogeneity
- $K$ agents, each with utility $u_k(x_1, \ldots, x_{N-1}) + x_N$
- Social welfare function under utilitarian criterion:

$$W(t) = \max_x \sum_{k=1}^{K} [u_k(x_1^k, \ldots, x_{N-1}^k) + Z]$$

$$- (p_1 + t)x_1^k - \sum_{i=2}^{N-1} p_i x_i^k \right] + \sum_{k=1}^{K} tx_1^k$$

- Structural method: estimate demand systems for all agents
- Sufficient statistic formula is unchanged—still need only slope of aggregate demand $\frac{dx_1}{dt}$

$$\frac{dW}{dt} = - \sum_{k=1}^{K} x_1^k + \sum_{k=1}^{K} x_k^k + t \frac{d}{dt} \sum_{k=1}^{K} x_1^k = t \frac{dx_1}{dt}$$
Discrete Choice Model

- Harberger sufficient statistic also works with discrete choice
- Agents have value $V_k$ for good 1; can either buy or not buy
- Let $F(V)$ denote distribution of valuations
- With 2 goods, utility of agent $k$ is
  $$V_k x_1 + Z - (p + t) x_1$$
- Social welfare:
  $$W(t) = \left\{ \int_{V_k} \max_{x_1^k} [V_k x_1^k + Z - (p_1 + t) x_1^k] dF(V_k) \right\}$$
  $$+ \int_{V_k} t x_1^k dF(V_k)$$
- This problem is not smooth at individual level, so cannot directly apply envelope thm. as stated
Discrete Choice Model

- Recast as planner’s problem choosing threshold above which agents are allocated good 1:

\[
W(t) = \max_{V} \int_{V}^{\infty} [V_k - (p_1 + t)] dF(V_k) + Z
\]

\[
+ \int_{V}^{\infty} dF(V_k)
\]

- Again obtain Harberger formula as a fn of slope of aggregate demand curve \(\frac{dx_1}{dt}\):

\[
\frac{dW}{dt} = -\left(1 - F(\bar{V})\right) + \left(1 - F(\bar{V})\right) + t \frac{d}{dt} \int_{V}^{\infty} dF(V_k)
\]

\[
\Rightarrow \frac{dW}{dt} = t \frac{dx_1}{dt}
\]
Deadweight loss is fully determined by difference between marginal willingness to pay for good $x_1$ and its cost ($p_1$)

- Recovering marginal willingness to pay requires an estimate of the slope of the demand curve because it coincides with marginal utility:

$$p = u'(x(p))$$

- Slope of demand is therefore sufficient to infer efficiency cost of a tax, without identifying rest of the model
Feldstein 1995, 1999

- Following Harberger, large literature in labor estimated effect of taxes on hours worked to assess efficiency costs of taxation.

- Feldstein observed that labor supply involves multiple dimensions, not just choice of hours: training, effort, occupation.

- Taxes also induce inefficient avoidance/evasion behavior.

- Structural approach: account for each of the potential responses to taxation separately and then aggregate.

- Feldstein’s alternative: elasticity of taxable income with respect to taxes is a sufficient statistic for calculating deadweight loss.
Feldstein Model: Setup

- Government levies linear tax $t$ on reported taxable income

- Agent makes $N$ labor supply choices: $l_1, \ldots, l_N$

- Each choice $l_i$ has disutility $\psi_i(l_i)$ and wage $w_i$

- Agents can shelter $e$ of income from taxation by paying cost $g(e)$

- Taxable Income ($TI$) is

$$TI = \sum_{i=1}^{N} w_i l_i - e$$

- Consumption is given by taxed income plus untaxed income:

$$c = (1 - t) TI + e$$
Agent’s utility is quasi-linear in consumption:

\[ u(c, e, l) = c - g(e) - \sum_{i=1}^{N} \psi_i(l_i) \]

Social welfare:

\[ W(t) = \{ (1 - t) TI + e - g(e) - \sum_{i=1}^{N} \psi_i(l_i) \} + t TI \]

Differentiating and applying envelope conditions for \( l_i \)

\( (1 - t) w_i = \psi'_i(l_i) \) and \( e \ (g'(e) = t) \) implies

\[ \frac{dW}{dt} = -TI + TI + t \frac{dT I}{dt} = t \frac{dT I}{dt} \]

Intuition: marginal social cost of reducing earnings through each margin is equated at optimum → irrelevant what causes change in \( TI \)
Simplicity of identification in Feldstein’s formula has led to a large literature estimating elasticity of taxable income.

But since primitives are not estimated, assumptions of model used to derive formula are never tested.

Chetty (2009) questions validity of assumption that $g'(e) = t$.

- Costs of some avoidance/evasion behaviors are transfers to other agents in the economy, not real resource costs.
- Ex: cost of evasion is potential fine imposed by government.
Individual chooses $e$ (evasion/shifting) and $l$ (labor supply) to

$$\max_{e,l} u(c, l, e) = c - \psi(l)$$

s.t. $c = y + (1 - t)(wl - e) + e - z(e)$

Social welfare is now:

$$W(t) = \{y + (1 - t)(wl - e) + e - z(e) - \psi(l)\} + z(e) + t(wl - e)$$

Difference: $z(e)$ now appears twice in SWF, with opposite signs.
Excess Burden with Transfer Costs

- Let $LI = wl$ be the total (pretax) earned income and $TI = wl - e$ denote taxable income.

- Exploit the envelope condition for term in curly brackets:

$$
\frac{dW}{dt} = -(wl - e) + (wl - e) + \frac{dz}{de} \frac{de}{dt} + t \frac{d[wl - e]}{dt}
$$

$$
= t \frac{dTI}{dt} + \frac{dz}{de} \frac{de}{dt}
$$

$$
= t \frac{dLI}{dt} - t \frac{de}{dt} + \frac{dz}{de} \frac{de}{dt}
$$

- First-order condition for individual’s choice of $e$:

$$
t = \frac{dz}{de}
$$

$$
\Rightarrow \quad \frac{dW}{dt} = t \frac{dLI}{dt} \quad (1)
$$

- Intuition: MPB of raising $e$ by $1$ (saving $t$) equals MPC.
Chetty (2009) Formula

- With both transfer cost $z(e)$ and resource cost $g(e)$ of evasion:

\[
\frac{dW}{dt} = t \frac{dLI}{dt} - g'(e) \frac{de}{dt}
\]

\[
= t \{ \mu \frac{dTl}{dt} + (1 - \mu) \frac{dLI}{dt} \}
\]

\[
= -\frac{t}{1-t} \{ \mu Tl \epsilon_{TI} + (1 - \mu) w l \epsilon_{LI} \}
\]

- $EB$ depends on weighted average of taxable income ($\epsilon_{TI}$) and total earned income elasticities ($\epsilon_{LI}$)

  - Practical importance: even though reported taxable income is highly sensitive to tax rates for rich, efficiency cost may not be large!

- Most difficult parameter to identify: weight $\mu$, which depends on marginal resource cost of sheltering, $g'(e)$
Estimate $\varepsilon_{LI}$ and $\varepsilon_{TI}$ to implement formula that permits transfer costs.

Insight: consumption data can be used to infer $\varepsilon_{LI}$

Estimate effect of 2001 flat tax reform in Russia on gap between taxable income and consumption, which they interpret as evasion.
Marginal personal income tax rate before and after the reform

Source: Gorodnichenko, Martinez-Vazquez, and Peter 2009
Consumption-income gap dynamics

Source: Gorodnichenko, Martinez-Vazquez, and Peter 2009
Taxable income elasticity $\frac{dTl}{dt}$ is large, whereas labor income elasticity $\frac{dLl}{dt}$ is not.

→ Feldstein’s formula overestimates the efficiency costs of taxation relative to more general measure for “plausible” $g'(e)$.

Question: could $g'(e)$ be estimated from consumption data itself?
Study deadweight cost from taxing diesel fuels, focusing on evasion.

Diesel fuel used for business purposes (e.g. trucking) is taxed, but residential purposes (e.g. heating homes) is not.

Substantial opportunity to evade tax.

1993: government added red dye to residential diesel fuel.

- Easy to monitor cheating by opening gas tank of a truck.

First document effect of dye reform on evasion.
Source: Marion and Muehlleger 2008
- Use reform to assess deadweight costs of evasion and taxation
  - Harder to evade → elasticity of behavior with respect to tax is much lower after reform

- Estimate price and tax elasticities before and after reform
  - Use cross-state variation in tax rates and price variation from world market
  - Note different interpretation of difference between price and tax elasticities in this study relative to tax salience papers
Price and Tax Elasticities By Year

Source: Marion and Muehlleger 2008
Elasticities imply that 1% increase in tax rate raised revenue by 0.60% before dye reform vs. 0.71% after reform.

Reform reduced deadweight cost of diesel taxation:

- $MDWL = 40$ cents per dollar of revenue raised before dye reform
- $MDWL = 30$ cents per dollar after reform

Lesson: Deadweight cost depends not just on preferences but also on enforcement technology.

But again need to think carefully about marginal costs of evasion in this context: social or transfer?
Welfare Analysis in Behavioral Models

- Formulas derived thus far rely critically on full optimization by agents in private sector

- How to calculate efficiency costs when agents do not optimize perfectly?

- Relates to broader field of behavioral welfare economics

- Focus on two papers here:

  1. Conceptual Issues: Bernheim and Rangel 2009
Abstractly, effect of policies on welfare are calculated in two steps

1. Effect of policy on behavior
2. Effect of change in behavior on utility

Challenge: identifying (2) when agents do not optimize perfectly

- How to measure objective function without tools of revealed preference?
- Danger of paternalism
Behavioral Welfare Economics: Two Approaches

- **Approach #1:** Build a positive model of deviations from rationality
  - Ex: hyperbolic discounting, bounded rationality, reference dependence
  - Then calculate optimal policy within such models

- **Approach #2:** Choice-theoretic welfare analysis (Bernheim and Rangel 2009)
  - Do not specify a positive model to rationalize behavior
  - Instead map directly from observed choices to statements about welfare
  - Analogous to “sufficient statistic” approach
Consider three different medicare plans with different copays: \( L, M, H \) and corresponding variation in premiums.

We have data from two environments:

1. On red paper, \( H > M > L \)
2. On blue paper, \( M > H > L \)
Behavioral Welfare Economics: Two Approaches

- Approach 1: build a model of why color affects choice and use it to predict which choice reveals “true” experienced utility

- Approach 2: Yields bounds on optimal policy
  - \( L \) cannot be optimal given available data irrespective of positive model
  - Optimal copay bounded between \( M \) and \( H \)

- Key insight: no theory of choice needed to make statements about welfare
  - Do not need to understand why color affects choice
Derive bounds on welfare based purely on choice data

In standard model, agents choose from a choice set \( x \in X \)

Goal of policy is to identify optimal \( x \)

In behavioral models, agents choose from “generalized choice sets” \( G = (X, d) \)

\( d \) is an “ancillary condition” – something that affects choice behavior but (by assumption) does not affect experienced utility

Ex: color of paper, salience, framing, default option
Let $C(X, d)$ denote choice made in a given GCS.

Choice inconsistency if $C(X, d) \neq C(X, d')$.

Define revealed preference relation $P$ as $xPy$ if $x$ always chosen over $y$ for any $d$.

Using $P$, can identify choice set that maximizes welfare instead of single point.

With continuous choices, effectively obtain bounds on welfare.
Consider a change in choice set from $X$ to $X' \subset X$

- Compute CV as amount needed to make agent indifferent to restriction of choice set for each $d$ (standard calculation)
- Lower bound on CV is minimum over all $d$’s
- Upper bound on CV is maximum over all $d$’s
Ex: suppose insurance plans are restricted to drop $M$ option

Under red paper condition, CV is 0 – no loss in welfare

Under blue paper condition, calculate price cut $\$z$ on $H$ needed to make agent indifferent between $M$ and $H$.

Bounds on CV: $(0, z)$

If $L$ option is dropped, bounds collapse to a singleton: $CV = 0$. 
Problem: looseness of bounds

Bounds tight when ancillary conditions do not lead to vast changes in choices

That is, bounds tight when behavioral problems are small

In cases where behavioral issues are important, this is not going to be a very informative approach
Solution: “refinements” – discard certain d’s as being “contaminated” for welfare analysis

- E.g. a neuroscience experiment shows that decisions made under red paper condition are more rational
- Or assume that choice rational when incentives are more salient

With fewer d’s, get tighter bounds on welfare and policy

Identifying “refinements” typically requires some insight into positive theory of behavior
Chetty, Looney, and Kroft (2009) section 5

- Derive partial-equilibrium formulas for incidence and efficiency costs
- Focus here on efficiency cost analysis
- Formulas do not rely on a specific positive theory, in the spirit of Bernheim and Rangel (2009)
Welfare Analysis with Salience Effects: Setup

- Two goods, \(x\) and \(y\); price of \(y\) is 1, pretax price of \(x\) is \(p\).

- Taxes: \(y\) untaxed. Unit sales tax on \(x\) at rate \(t^S\), which is not included in the posted price.

- Tax-inclusive price of \(x\): \(q = p + t^S\)
Representative consumer has wealth $Z$ and utility $u(x) + v(y)$

Let $\{x^*(p, t^S, Z), y^*(p, t^S, Z)\}$ denote bundle chosen by a fully-optimizing agent

Let $\{x(p, t^S, Z), y(p, t^S, Z)\}$ denote empirically observed demands

Place no structure on these demand functions except for feasibility:

$$(p + t^S)x(p, t^S, Z) + y(p, t^S, Z) = Z$$
Price-taking firms use $y$ to produce $x$ with cost fn. $c$

Firms optimize perfectly. Supply function $S(p)$ defined by:

$$p = c'(S(p))$$

Let $\varepsilon_S = \frac{\partial S}{\partial p} \times \frac{p}{S(p)}$ denote the price elasticity of supply
Define excess burden using EV concept

Excess burden (EB) of introducing a revenue-generating sales tax $t$ is:

$$EB(t^S) = Z - e(p, 0, V(p, t^S, Z)) - R(p, t^S, Z)$$
Preference Recovery Assumptions

**A1** Taxes affect utility only through the chosen consumption bundle. Agent’s indirect utility given tax of $t^S$ is

$$V(p, t^S, Z) = u(x(p, t^S, Z)) + v(y(p, t^S, Z))$$

**A2** When tax inclusive prices are fully salient, the agent chooses the same allocation as a fully-optimizing agent:

$$x(p, 0, Z) = x^*(p, 0, Z) = \arg \max_x u(x) + v(Z - px)$$

- A1 specifies ancillary condition: tax rate and salience does not enter utility directly
- A2 is a refinement: behavior when tax is salient reveals true preferences
Two demand curves: price-demand $x(p, 0, Z)$ and tax-demand $x(p_0, t^S, Z)$

Two steps in efficiency calculation:

1. Use price-demand $x(p, 0, Z)$ to recover utility as in standard model
2. Use tax-demand $x(p, t^S, Z)$ to calculate $V(p, t^S, Z)$ and EB
Excess Burden with No Income Effect for Good $x \times \left( \frac{\partial x}{\partial Z} = 0 \right)$

$E_B \approx -\frac{1}{2}(t^S)^2 \frac{\partial x}{\partial \epsilon p} \frac{\partial x}{\partial t^S}$

Source: Chetty, Looney, and Kroft (2009)
Efficiency Cost: No Income Effects

- Without income effects \( \frac{\partial x}{\partial Z} = 0 \), excess burden of introducing a small tax \( t^S \) is

\[
EB(t^S) \approx -\frac{1}{2}(t^S)^2 \frac{\partial x}{\partial t^S} \frac{\partial x}{\partial p} \frac{\partial x}{\partial p} = \frac{1}{2} (\theta t^S)^2 x(p, t^S, Z) \frac{\epsilon_D}{p + t^S}
\]

- Inattention reduces excess burden when \( d x / d Z = 0 \).

- Intuition: tax \( t^S \) induces behavioral response equivalent to a fully perceived tax of \( \theta t^S \).

- If \( \theta = 0 \), tax is equivalent to a lump sum tax and \( EB = 0 \) because agent continues to choose first-best allocation.
Efficiency Cost with Income Effects

- Same formula, but all elasticities are now compensated:

\[ EB(t^S) \approx -\frac{1}{2} (t^S)^2 \frac{\partial x^c / \partial t^S}{\partial x^c / \partial p} \frac{\partial x^c / \partial t^S}{\partial x^c / \partial p} \]

\[ = \frac{1}{2} (\theta^c t^S)^2 x(p, t^S, Z) \frac{\varepsilon_D}{p + t^S} \]

- Compensated price demand: \( dx^c / dp = dx / dp + xdx / dZ \)

- Compensated tax demand: \( dx^c / dt^S = dx / dt^S + xdx / dZ \)

- Compensated tax demand does not necessarily satisfy Slutsky condition \( dx^c / dt^S < 0 \) b/c it is not generated by utility maximization
Efficiency Cost with Income Effects

\[ EB(t^S) \approx -\frac{1}{2} (t^S)^2 \frac{\partial x^c / \partial t^S}{\partial x^c / \partial p} \frac{\partial x^c / \partial t^S}{\partial x^c / \partial p} \]

\[ = \frac{1}{2} (\theta^c t^S)^2 x(p, t^S, Z) \frac{\epsilon^c_D}{p + t^S} \]

- With income effects \((dx/dZ > 0)\), making a tax less salient can raise deadweight loss.

- Tax can generate \(EB > 0\) even if \(dx/dt^S = 0\)

- Example: consumption of food and cars; agent who ignores tax on cars underconsumes food and has lower welfare.

- Intuition: agent does not adjust consumption of \(x\) despite change in net-of-tax income, leading to a positive compensated elasticity.
Directions for Further Work on Behavioral Welfare Analysis

1. Normative analysis of tax policy
   - Value of tax simplification
   - Tax smoothing

2. Use similar approach to welfare analysis in other contexts
   - Design consumer protection laws and financial regulation in a less paternalistic manner by studying behavior in domains where incentives are clear